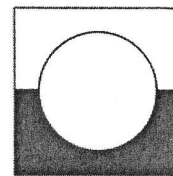


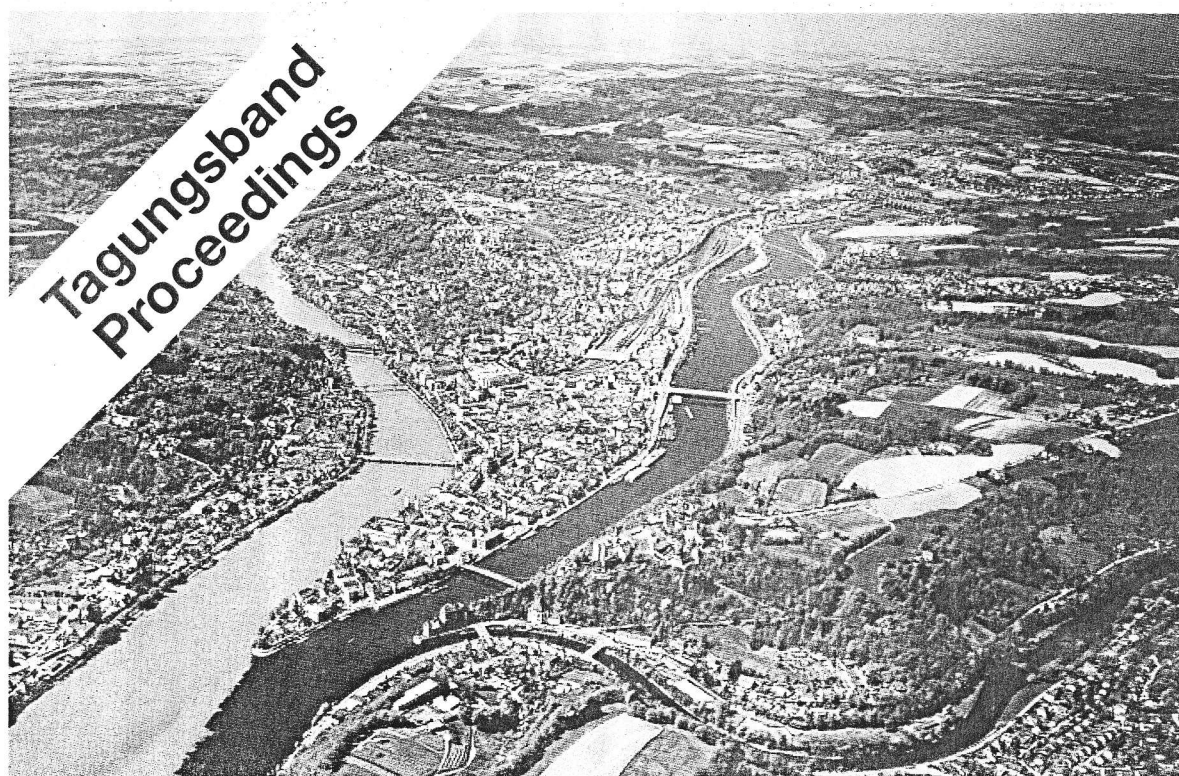
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Stability Analysis of Slopes with Translational Failure Surfaces and Steady Seepage Flow

Standsicherheit von Böschungen mit Translationsgleitfugen und stationärer Sickerströmung

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ABSTRACT: *There are simple analytical solutions for stability analysis of translation slides with failures along essentially planar surfaces via limit equilibrium methods. The solutions for special cases with flow either horizontal or parallel to the surface are usually given for the conditions of steady seepage flow. The general solutions for the stress and analysis of gravitational stability of slopes with infinite and finite height are presented in the paper, with reference to the hydraulic stability, for an arbitrary direction of filtration in statical and seismic (quasi-static) conditions. The solutions (analytical and chart diagrams) are given for the infinite slope for the special cases in homogenous and two-layer soil, which is expected at earthfill dams. The geometrical conditions for limit equilibrium of the soil with given characteristics are presented for slope with finite height.*

ZUSAMMENFASSUNG: *Für die Standsicherheitsanalyse von Böschungen mit Translationsgleitfugen bestehen die einfache analytische Lösungen, die mittels Methode des Grenzgleichgewicht zu erreichen sind. In der Literatur sind üblicherweise die Lösungen für Spezialfälle mit stationärer Strömung in vertikalen und horizontalen Richtung, oder parallel mit der Böschungsneigung gegeben. In dieser Veröffentlichung sind die allgemeine Lösungen für Spannungen und Wertschätzung der Gravitationsstabilität der Böschungen von unbegrenzten und begrenzten Höhe, mit dem Rückblick auf die hydraulische Stabilität, für die beliebige gerichteten lineare Strömung unter statischen und seismischen (quasi-statischen) Bedingungen vorgelegt. Für die Böschungen der unbegrenzten Höhe die Lösungen für fünf spezielle Fällen in gleichförmigem und zwei-schichtigem Boden, wie in Aufschüttung erwartet, sind gegeben.*

Introduction

The two-dimensional analysis of the gravitational stability of the slopes with infinite and finite height for planar slip surfaces has in practice the simple analytical solutions done by the limit equilibrium method for the rigid-plastic material and Terzaghi-Mohr-Coulomb failure condition for the effective normal and (total) shearing stresses in the soil. The influences of soil water seepage change the effective stresses (pore water pressures) and slope safety factor and the solutions become more complex so in literature are given even for rectilinear flow for special or critical cases only.

The solutions can be derived through the average stresses in the slip plane, determined

from the equilibrium conditions of the whole prismatic sliding element (1) or the rectangular element in the slipping zone (2). For each approach, the stresses can be determined in two ways (Fig.1): through the total gravitational force (W) and the pore water pressure force (U) in the slip plane or through the effective gravitational force (W') and the seepage force (J) – as the resulting volume force (W'') is determined in such two ways out of possible three (1). The first way is more frequent and more suitable for in the practice it is usually easier to determine the resultant of pore pressure than the seepage force. The second way displays the effect of filtration better and enables the derivation of the hydraulic stability conditions of cohesionless soil from $N'=0$ for the analysed case.

For conventional calculation, we considered useful the derivation (in both ways) and the application of 'general' solutions for the safety factor of the infinite and finite height slopes with the planar slip surfaces and rectilinear flow in the seismic (quasi-static) conditions, where the solutions for the particular cases are presented. Of course, their application is limited by the high level of introduced geometrical and physical simplifications (reduction to the twodimensional problem, linear failure condition without deformations, pseudo-static seismic analysis etc.).

Finite height slope

The solution for the safety factor with the classic failure condition along the slip plane is given in the following text, with the equilibrium polygons of forces in non-seismical and seismical conditions (Fig.1).

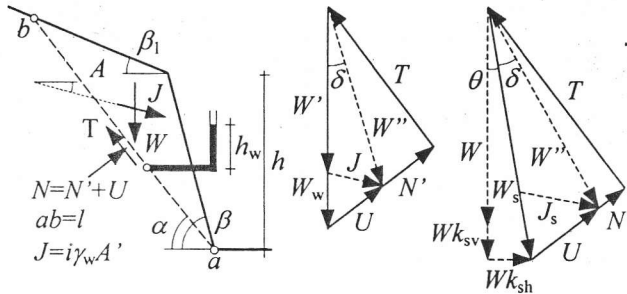


Fig.1: Forces polygons for the finite height slope.

$$l = h \sin(\beta - \beta_1) / [\sin \beta \sin(\alpha - \beta_1)]$$

$$A = \frac{h}{2} \sin(\beta - \alpha) \sin(\beta - \beta_1) / [\sin^2 \beta \sin(\alpha - \beta_1)]$$

$$l/A = (2/h) \sin \beta / \sin(\beta - \alpha)$$

$$U = \gamma_w \int h_w dl, \quad h_w = f(\alpha, \beta, \dots)$$

$$W = \sum \gamma_i A_i = \bar{\gamma} A$$

$$\theta = \tan^{-1} [k_{sh} / (1 + k_{sv})]$$

$$k_s = \sqrt{(1 + k_{sv})^2 + k_{sh}^2}$$

$$W_s = W k_s$$

$$N = W_s \cos(\alpha + \theta)$$

$$T = W_s \sin(\alpha + \theta)$$

$$F_s = \frac{c'l + (N - U) \tan \varphi'}{T} = - \frac{U \tan \varphi'}{W_s \sin(\alpha + \theta)} + \frac{c'}{\bar{\gamma} h k_s \sin(\alpha + \theta) \sin(\beta - \alpha)} + \frac{\tan \varphi'}{\tan(\alpha + \theta)}$$

For the known distribution of pore pressures along the slip plane the first member of the expression (U/W_s) can be expressed through the

geometrical characteristics of the problem. For the known force of pore pressures (U) the safety factor not depends of the ground surface inclination (β_1), as shown by Taylor (1) for the static conditions without seepage.

For the limit equilibrium ($F_s=1$) without seepage and seismic influences:

$$\frac{\gamma h}{c'} = \frac{2 \sin \beta \cos \varphi'}{\sin(\beta - \alpha) \sin(\alpha - \varphi')}$$

$$\left(\frac{\gamma h}{c'} \right)_{\alpha} = 0: \alpha_{cr} = \frac{\beta + \varphi}{2}, \quad h_{cr} = \frac{4c'}{\gamma} \frac{\sin \beta \cos \varphi'}{1 - \cos(\beta - \varphi')}$$

$$\beta = \frac{\pi}{2}: \quad \alpha_{cr} = \frac{\pi}{4} + \frac{\varphi}{2}, \quad h_{cr} = \frac{4c'}{\gamma} \tan\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)$$

The relations are valid for the effective or total stresses along the slip plane (withouth the tension crack in the fine graded soil) and for the simplifications seen in the derivation procedure. In Fig.2 they are presented in the form suitable for comparison with the classic solution for the total stresses along slip circle (1,3).

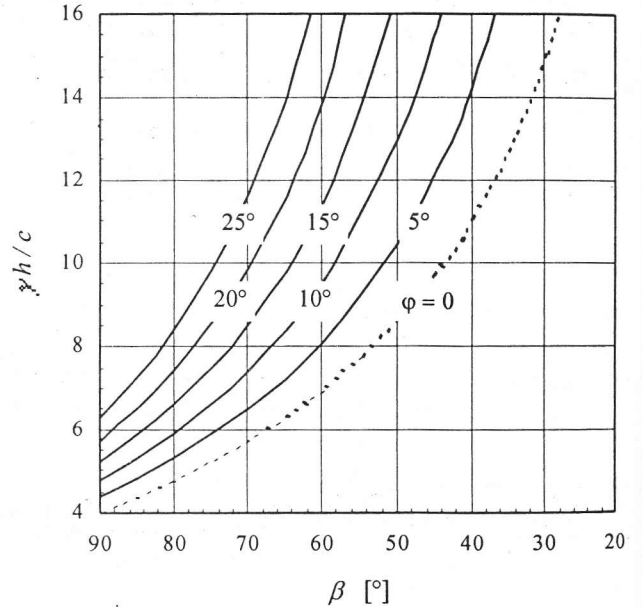


Fig.2: Statical limit equilibrium condition for the finite height slope without seepage.

Infinite height slope

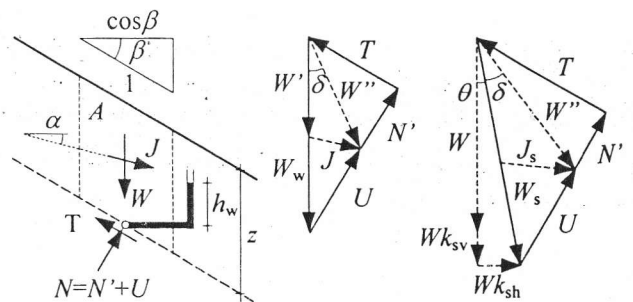
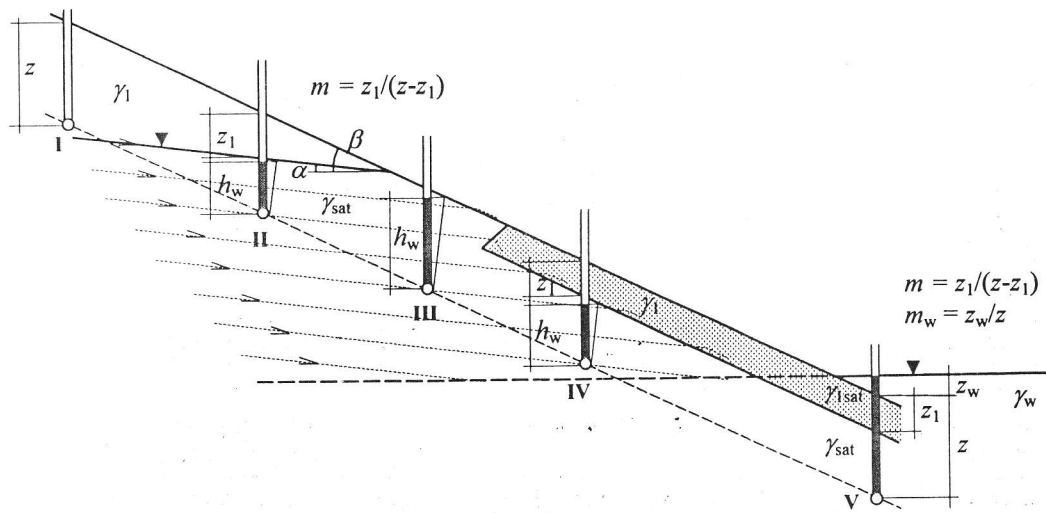


Fig.3: Infinite height slope force polygons.



α		I ($z_1=z$)	II ($0 < z_1 < z$)	III (IV: $z_1=0$)	IV ($0 < z_1 < z$)
	σ_v	$z\gamma_1$	$z(\gamma_1 m + \gamma_{sat})/(1+m)$	$z\gamma_{sat}$	$z(\gamma_1 m + \gamma_{sat})/(1+m)$
	i	0	$\sin \alpha$	$\sin \beta / \cos(\beta - \alpha)$	$\sin \beta / \cos(\beta - \alpha)$
	u	0	$z\gamma_w / [(1 + \tan^2 \beta)(1+m)]$	$z\gamma_w / (1 + \tan \beta \tan \alpha)$	$z\gamma_w / [(1 + \tan \beta \tan \alpha)(1+m)]$
α	r_u	0	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \frac{1}{1 + \tan^2 \alpha}$	$\frac{\gamma_w}{\gamma_{sat}} \frac{1}{1 + \tan \beta \tan \alpha}$	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \frac{1}{1 + \tan \beta \tan \alpha}$
	k	1	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \frac{1 + \tan^2 \beta}{1 + \tan^2 \alpha}$	$1 - \frac{\gamma_w}{\gamma_{sat}} \frac{1 + \tan^2 \beta}{1 + \tan \beta \tan \alpha}$	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \frac{1 + \tan^2 \beta}{1 + \tan \beta \tan \alpha}$
$\pm \pi/2$	r_u	0	—	0	0
	k	1	—	1	1
$\beta \pm \pi/2$	r_u	0	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \sin^2 \beta$	∞	∞
	k	1	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \tan^2 \beta$	$-\infty$	$-\infty$
0	r_u	0	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}}$	$\frac{\gamma_w}{\gamma_{sat}}$	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}}$
	k	1	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} (1 + \tan^2 \beta)$	$1 - \frac{\gamma_w}{\gamma_{sat}} (1 + \tan^2 \beta)$	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} (1 + \tan^2 \beta)$
β	r_u	0	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \cos^2 \beta$	$\frac{\gamma_w}{\gamma_{sat}} \cos^2 \beta$	$\frac{\gamma_w}{\gamma_1 m + \gamma_{sat}} \cos^2 \beta$
	k	1	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}}$	$1 - \frac{\gamma_w}{\gamma_{sat}} \cong \frac{1}{2}$	$1 - \frac{\gamma_w}{\gamma_1 m + \gamma_{sat}}$

	V ($0 < z_1 < z$)	V ($z_1=z$)	V ($z_1=0$)
σ_v	$z[\gamma_w m_w + (\gamma_{1sat} m + \gamma_{sat})/(1+m)]$	$z(\gamma_w m_w + \gamma_{1sat})$	$z(\gamma_w m_w + \gamma_{sat})$
i	0		
u	$z\gamma_w(1+m_w)$		
r_u	$\frac{1+m_w}{m_w + \frac{\gamma_{1sat} m + \gamma_{sat}}{\gamma_w(1+m)}}$	$\frac{1+m_w}{m_w + \frac{\gamma_{1sat}}{\gamma_w}}$	$\frac{1+m_w}{m_w + \frac{\gamma_{sat}}{\gamma_w}}$
k	$1 - \frac{(1+m_w)(1+\tan^2 \beta)}{m_w + \frac{\gamma_{1sat} m + \gamma_{sat}}{\gamma_w(1+m)}}$	$1 - \frac{(1+m_w)(1+\tan^2 \beta)}{m_w + \frac{\gamma_{1sat}}{\gamma_w}}$	$1 - \frac{(1+m_w)(1+\tan^2 \beta)}{m_w + \frac{\gamma_{sat}}{\gamma_w}}$

$$\sigma_v = \sum \gamma_i \Delta z_i$$

$$u = \gamma_w h_w, \quad r_u = u / \sigma_v$$

$$\sigma = \sigma_v \cos^2 \beta$$

$$\sigma' = \sigma_v (\cos^2 \beta - r_u)$$

$$\tau = \sigma_v \cos \beta \sin \beta$$

$$k = 1 - r_u / \cos^2 \beta$$

$$F_s = \frac{2c'}{\sigma_v \sin 2\beta} + k \frac{\tan \phi'}{\tan \beta}$$

$$\tan \delta = \tan \beta / (1/r_u - 1)$$

$$\tan(\beta + \delta) = \tan \beta / k$$

Fig.4: Statical solutions for particular cases I-V of the infinite slope with the linear seepage.

$$\sigma_v = \sum_0^z \gamma_i \Delta z_i = \bar{\gamma} z$$

$$u = \gamma_w h_w, \quad h_w = f(\alpha, \beta, \dots)$$

$$r_u = u / \sigma_v$$

$$W = W' + W_w = \sigma_v \cos \beta$$

$$\theta = \tan^{-1} [k_{sh} / (1 + k_{sv})]$$

$$k_s = \sqrt{(1 + k_{sv})^2 + k_{sh}^2}$$

$$W_s = W k_s$$

$$\sigma = W_s \cos(\beta + \theta) = \sigma_v k_s \cos \beta \cos(\beta + \theta)$$

$$\sigma' = \sigma - u = \sigma_v [k_s \cos \beta \cos(\beta + \theta) - r_u]$$

$$\tau = W_s \sin(\beta + \theta) = \sigma_v k_s \cos \beta \sin(\beta + \theta)$$

$$k = 1 - r_u / [k_s \cos \beta \cos(\beta + \theta)]$$

$$F_s = \frac{c'}{\sigma_v k_s \cos \beta \sin(\beta + \theta)} + k \frac{\tan \varphi'}{\tan(\beta + \theta)}$$

$$\tan(\beta + \theta + \delta) = \tan(\beta + \theta) / k$$

$$\tan \delta = \tan(\beta + \theta) / \{k_s \cos \beta / [r_u \cos(\beta + \theta)] - 1\}$$

Limit values of certain parameters can be determined from the limit equilibrium ($F_s=1$):

$$\frac{c'}{\sigma_v} = k_s \cos \beta \cos(\beta + \theta) [\tan(\beta + \theta) - \tan \varphi] + r_u \tan \varphi'$$

For $c'=0$:

$$r_u = k_s \cos \beta \cos(\beta + \theta) [1 - \tan(\beta + \theta) / \tan \varphi']$$

$$\tan \varphi' = \tan(\beta + \theta + \delta)$$

and without seismic influences:

$$r_u = \cos^2 \beta (1 - \tan \beta / \tan \varphi')$$

$$\tan \varphi' = \tan(\beta + \delta)$$

The analysis of conditions where the critical slip plane is parallel to the slope, as well as the comparison of pseudo-static and dynamical approach can be seen in (3). Bear in mind that the quasi-static approach does not introduce the increase of pore pressure due to the seismic influences, that $W k_{sh}$ acts short and not continuously and so on.

From the solution for the general case (Fig.3), on the Fig.4 are the solutions for the five frequent particular cases (I-V) with the linear seepage without seismic influences.

These solutions contain all the solutions for simple cases, available in literature in the different form, for the non-coherent soil for the cases II (4), III (5,6) and IV (5) or for a coherent soil and seepage parallel to the slope in cases II (7,8) III (3) and IV (3,9).

For the illustration of possibilities in searching the required values of certain parameters for the

limit equilibrium ($F_s=1$) of the slope, chart for the case III in non-coherent soil is presented (Fig.5).

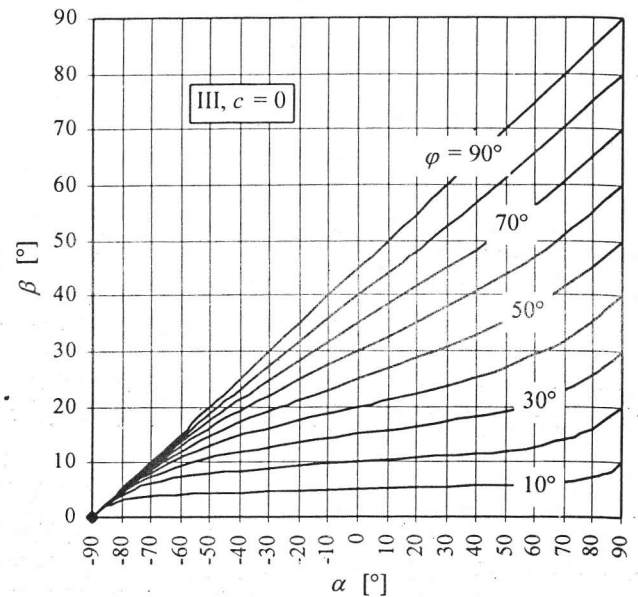


Fig.5: The limit equilibrium condition for case III in the non-coherent soil ($c'=0$).

The literature (3,10) contains for practice attractive but discarded charts (Hoek, 1970 and later) with the solutions for plane failure in finite height rock slopes. The approach for the solution of linear seepage and slip circle (6) is also interesting, which is beyond the subject of this paper.

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